Calculus Warm Up # 8-2

From Alg 2 and PC...

$A = Pe^{rt}$

1. For a savings account in which interest is compounded continuously, find the following for an initial investment of $20,000 at an annual interest rate of 10.5%.

a) time to double

b) balance after 10 yrs.

HW Questions: p. 399

1. $\int \frac{1}{x+1}\,dx$

3. $\int \frac{1}{3-2x}\,dx$

5. $\int \frac{x}{x^2+1}\,dx$

7. $\int \frac{x^2 - 4}{x}\, dx$

9. $\int_1^e \frac{\ln x}{2x}\, dx$ let $u = \ln x$ $du = \frac{1}{x}dx$

11. $\int_1^e \frac{(1 + \ln x)^2}{x}\, dx$

13. $\int_0^2 \frac{x^2 - 2}{x + 1}\, dx$

$$\begin{array}{r|rrr} -1 & 1 & 0 & -2 \\ & & -1 & 1 \\ \hline & 1 & -1 & -1 \end{array}$$

$x - 1 - \frac{1}{x+1}$

15. $\int \frac{1}{\sqrt{x} + 1}\, dx$

17. $\int \frac{x^2 + 2x + 3}{x^3 + 3x^2 + 9x}\, dx$

19. $3\int \frac{1}{3x^{2/3}(1+x^{1/3})}\,dx$

21. $\int \frac{1}{1+\sqrt{x}}\,dx$

23. $\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$

Let $u = 1 + x^{1/3}$

$du = \frac{1}{3}x^{-2/3}$

$= \frac{1}{3x^{2/3}}\,dx$

$3\int \frac{1}{u}\,du$

Let $u = 1 + \sqrt{x} \to \sqrt{x} = u - 1$

$du = \frac{1}{2\sqrt{x}}\,dx \to dx = 2\sqrt{x}\,du$

$\int \frac{1}{u}\cdot 2\sqrt{x}\,du$

$2\int \frac{1}{u}(u-1)\,du$

$2\int 1\,du - 2\int \frac{1}{u}\,du$

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19. $\int \frac{1}{x^{2/3}(1+x^{1/3})}\,dx$

21. $\int \frac{1}{1+\sqrt{x}}\,dx$

23. $\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$

Let $u = \sqrt{x} - 3 \to (\sqrt{x})^2 = (u+3)^2$

$du = \frac{1}{2\sqrt{x}}\,dx \to dx = 2\sqrt{x}\,du$

$\int \frac{1}{u}\cdot\sqrt{x}\cdot 2\sqrt{x}\,du$

$2\int \frac{(u+3)^2}{u}\,du$

25. $-\frac{2}{3}\int \frac{-3\sqrt{x}}{2(1 - x\sqrt{x})}\,dx$

27. $\int \frac{x(x-2)}{(x-1)^3}\,dx$

$u = 1 - x^{3/2}$

$du = -\frac{3}{2}x^{1/2}$

$-\frac{2}{3}\int \frac{1}{u}\,du$

let $u = x - 1$

$x = u + 1$

Remember differentiation rule:

$$\frac{d}{dx}\left[a^u\right] = (\ln a)\, a^u\, du$$

We need (ln a) in the integrand:

$$\int a^u\,du = \frac{1}{\ln a}\int (\ln a)\, a^u\,du$$

$$= \frac{1}{\ln a}\cdot a^u + C$$

$$\boxed{= \frac{a^u}{\ln a} + C}$$

29. $\int 3^x \, dx$

31. $\int_{-1}^{2} 2^x \, dx$

33. $\frac{1}{2}\int 2x5^{x^2} \, dx$ let $u = x^2$ $du = 2x\,dx$

$\frac{1}{2}\int 5^u \, du$

$\frac{1}{2} \cdot \frac{1}{\ln 5}\int (\ln 5)\, 5^u \, du$

$\frac{1}{2\ln 5} \cdot 5^{x^2} + C$

$\boxed{\frac{5^{x^2}}{\ln 25} + C}$

7.7

Growth and Decay

Newton's Law of Cooling

Law of Exponential Growth and Decay

Let y = a differentiable function of time, t.

and the rate of change in y over time is directly proportional to y:

$$\frac{dy}{dt} = ky$$ k = constant of proportionality

$$\int \frac{1}{y}\,dy = \int k\,dt$$

Solve the differential equation for y using the technique:

Separation of Variables

$$\ln y = kt + C$$

$$y = e^{kt + C}$$

$$y = e^{kt} \cdot e^{C}$$ just a constant

$$\boxed{y = Ce^{kt}}$$

$$y = Pe^{rt}$$

Law of Exponential Growth and Decay

from $\frac{dy}{dt} = ky \longrightarrow y = Ce^{kt}$

C = Initial Amount
k = constant of proportionality

Exponential Growth for k > 0

Exponential Decay for k < 0

Like our compound Interest Formula:

$$y = Pe^{rt}$$

P = Initial Amount
r = interest rate
(constant of proportionality)

Basic steps for solving the differential equation using

Separation of Variables

1) Separate
2) Integrate
3) Find C
4) Find k
5) Find what answers the question

Newton's Law of Cooling

The rate of change in temperature of an object is proportional to the difference in the temperature of the object and the temperature of the surrounding medium.

Let y = the temperature of an object

T = temperature of surrounding medium

t = time

$$\frac{dy}{dt} = k(y - T)$$

A room is kept at a constant temperature of 60° F. If an object cools from 100° to 90° in 10 minutes, how much longer will it take for its temperature to decrease to 80° ?

given: (t, y)

$\frac{dy}{dt} = k(y - 60)$ $(0, 100°)$ & $(10, 90°)$

$\frac{1}{y-60}\,dy = k\,dt$ separate the variables

$\int \frac{1}{y-60}\,dy = \int k\,dt$ integrate

$\ln(y-60) = kt + C$ (no ab.val. b/c y > 60)

$\ln(100-60) = k(0) + C$ find C with (0, 100)

$C = \ln 40$

$\ln(y-60) = kt + \ln 40$

$kt = \ln\left(\frac{y-60}{40}\right)$

$k(10) = \ln\left(\frac{90-60}{40}\right)$ find k with (10, 90)

$k = \frac{\ln(0.75)}{10}$

$\frac{\ln(0.75)}{10} \cdot t = \ln\left(\frac{80-60}{40}\right)$ plug in y = 80

$t = \frac{10\ln(0.5)}{\ln(0.75)}$

$t \approx 24.09$ minutes to cool to 80°

So $24.09 - 10 \approx 14.09$ additional minutes.

HW:

7.6 p. 399, # 37 - 45 odd

7.7 p. 404, # 9, 19, 21, 29 - 31

answer #30) ≈ 119.7 minutes

Let's start # 29 together...

29. Using Newton's Law of Cooling (see Example 5), determine the reading on a thermometer 5 minutes after it is taken from a room at 72° Fahrenheit to the outdoors where the temperature is 20°, if the reading dropped to 48° after 1 minute.

$t = 5$

$y = ?$

given (t, y) ⟶ $(0, 72°)$ & $(1, 48°)$

Separation of Variables

1) Separate
2) Integrate
3) Find C
4) Find k
5) Find what answers the question

$$\frac{dy}{dt} = k(y - T)$$

$$\int \frac{1}{y-20}\,dy = \int k\,dt$$

$$\ln(y-20) = kt + C$$

$$\ln(72-20) = k(0) + C \quad \leftarrow (0, 72°)$$

$$C = \ln 52$$

$$kt = \ln\left(\frac{y-20}{52}\right)$$

$$k(1) = \ln\left(\frac{48-20}{52}\right) \quad \leftarrow (1, 48°)$$

$$k = \ln\left(\frac{7}{13}\right)$$

$$\left[\ln\left(\frac{7}{13}\right)\right]t = \ln\left(\frac{y-20}{52}\right)$$

$$5\ln\left(\frac{7}{13}\right) = \ln\left(\frac{y-20}{52}\right)$$

$$\left(\frac{7}{13}\right)^5 = \frac{y-20}{52}$$

$$y = 52\left(\frac{7}{13}\right)^5 + 20$$

$$y \approx 22.35°$$